Hexadecapole interaction and the $\triangle I$=4 staggering effect in rotational bands

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Abstract

A role of the multipole interaction in the description of the $\triangle I$=4 staggering phenomenon is investigated in a model consisting of a single-j shell filled by identical nucleons. Exact diagonalization of the quadrupole-plus-hexadecapole Hamiltonian shows that the hexadecapole-hexadecapole interaction can produce a $\triangle I$=4 periodicity in the yrast sequence.
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1 Introduction

Recently a $\triangle I$=4 staggering effect in the dynamical moment of inertia of some superdeformed bands has been observed [1, 2, 3]. This can be interpreted as a bifurcation of the yrast band into two sequences with spins varying by 4 units within each sequence. The corresponding energy splitting is very small, i.e., about 100 eV. This effect may occur due to remnants of the C_4 symmetry of the system and suggests that presence of hexadecapole deformations or hexadecapole-type multipole interactions may be responsible for the staggering phenomenon. Recently, the origin of the staggering has been discussed by means of a phenomenological Hamiltonian containing higher-order terms in angular momentum [4]. In this work we investigate the effect by means of a more microscopic approach involving schematic multipole-multipole interactions.

2 Theoretical Model

The model to test the $\triangle I$=4 staggering phenomenon should be able to take into account the interplay between rotation and shape dynamics of a many-body system. A degenerate single-j shell occupied by identical nucleons [5, 6], interacting via quadrupole-quadrupole and hexadecapole-hexadecapole multipole forces has this property. The corresponding Hamiltonian may be written as

$$\hat{H} = -\chi_2 \hat{Q}_2 \cdot \hat{Q}_2 - \chi_4 \hat{Q}_4 \cdot \hat{Q}_4 \ , \tag{1}$$

where $\hat{Q}_2$ and $\hat{Q}_4$ are the quadrupole and hexadecapole moment operators defined as $\hat{Q}_{\lambda\mu}=\sum_{mm'}(jmjm'|\lambda\mu)a^+_m\tilde{a}_{m'}$ where $\tilde{a}_m \equiv T a_m T^{-1}$, T is the time reversal operator, and the dot symbol stands for the scalar product $\hat{Q}_\lambda \cdot \hat{Q}_\lambda = \sum_\mu (-)^\mu \hat{Q}_{\lambda\mu}\hat{Q}_{\lambda,-\mu}$. In the harmonic oscillator approximation [7] the values of coupling constants χ_2 and χ_4 can be estimated as

$$\chi_2 = \frac{176}{A} \text{ MeV} \ , \qquad \chi_4 = \frac{165}{A} \text{ MeV} \ . \tag{2}$$

The results of the diagonalization depend solely on the ratio χ_4/χ_2. Therefore, in the following, we assumed χ_2=1 MeV (i.e., χ_2 is used to fix energy scale), while the parameter χ_4, describing the relative strength of hexadecapole and quadrupole interactions, was varied to investigate the conditions for an appearance of the staggering effect.

In this paper we discuss the case of N=8 particles in the j=15/2 shell, although we have performed a series of calculations for several different shells and particle numbers. A choice of N=8 particles corresponds to a half-filled shell, and gives a regular collective yrast band. The calculations were performed for the parameter χ_4 ranging from χ_4=0 to χ_4=10 MeV. For larger values of χ_4, one only obtains an energy scaling corresponding to the dominating hexadecapole interaction.

To analyse a $\triangle I$=4 staggering effect in collective bands, we extracted the smooth reference curve according to Ref. [2]. Quantities $\triangle E_\gamma(I) \equiv E_\gamma(I+2) - E_\gamma(I)$ and $\triangle E^{\rm ref}_\gamma(I) \equiv [\triangle E_\gamma(I+2) + 2\triangle E_\gamma(I) + \triangle E_\gamma(I-2)]/4$ were obtained in this way. Then staggering parameter $\triangle E_\gamma(I) - \triangle E^{\rm ref}_\gamma(I)$ was evaluated and plotted in each case.

3 Results and Discussion

Fig. 1 shows a complete quadrupole spectrum of Hamiltonian (1) in case of χ_4=0. Solid lines connect states of spins I and I+2 with the largest reduced matrix elements of the quadrupole moment operator. Several quite regular rotational bands were obtained. The staggering parameter was evaluated for each case, but no $\triangle I$=4 periodicity was found. In Figs. 2a and 2b the collective energies $E(I)$ and the parameters $\triangle E_\gamma(I) - \triangle E^{\rm ref}_\gamma(I)$ of the yrast sequence are presented as open circles. When the hexadecapole interaction is switched on, the staggering $\triangle I$=4 appears. This is seen in Fig. 2b, where full circles denote the staggering parameters of the yrast band for χ_4=0.3, 0.4, and 0.5 MeV, while the corresponding yrast spectra are plotted in Fig. 2a. The staggering effect is small and therefore difficult to be directly seen in the yrast spectra.

The amplitude of the staggering increases with χ_4, but for the values of χ_4 larger than 0.5 MeV, the yrast states are no longer connected by the enhanced E2 transitions; i.e., the band structure is lost. Fig. 3a shows the yrast states for χ_4=4 MeV. This corresponds to the situation where the hexadecapole interaction dominates and the yrast sequence is strongly perturbed. There are no strong E2 transitions linking the yrast states, but the staggering parameter extracted from the energies of the yrast sequence exhibits the $\triangle I$=4 periodicity (Fig. 3b).

Similar effects can be obtained for other values of j and N. We would like to stress that it was impossible to obtain similar staggering by using the quadrupole-quadrupole interaction alone. On the other hand, for j=15/2 and N=8, the staggering appears only in the yrast band.

The examples presented here show that the hexadecapole interaction may generate the $\triangle I$=4 irregularities. The proper treatment of the hexadecapole interaction seems to be crucial for the understanding of this intriguing staggering phenomenon.

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Figure 1: The exact spectrum for the j=15/2 shell filled with N=8 particles. The coupling constants χ_2=1 MeV, χ_4=0 were used.

Figure 2: The results for the j=15/2 shell filled with N=8 particles. The quadrupole coupling constant was χ_2=1 MeV. The hexadecapole strengths χ_4=0 (open circles) and χ_4=0.3, 0.4, 0.5 MeV (full circles) were used. Part (a) shows the yrast spectra, part (b) the corresponding staggering parameters $\triangle E_\gamma(I) - \triangle E_\gamma^{\rm ref}(I)$ plotted as a function of spin.

Figure 3: The exact yrast spectrum (a) and the corresponding staggering parameter $\triangle E_\gamma(I) - \triangle E_\gamma^{\rm ref}(I)$ (b) for the j=15/2 shell filled with N=8 particles. The quadrupole and hexadecapole coupling constants were χ_2=1 MeV, χ_4=4 MeV, respectively.